STAT 5700 — Quiz 3
Date: September 18, 2025
SOLUTIONS

Problem 1

Suppose a small university has 2 dining halls that serve meals every day. Dining Hall A serves 60% of all student meals, and 5% of its meals are rated unsatisfactory. Dining Hall B serves 40% of the meals, and 2% of its meals are rated unsatisfactory. A meal is selected at random from the total meals served. If the meal is found to be rated unsatisfactory, what is the probability that it came from Dining Hall A?

SOLUTION

Note, tree diagram is useful

Given: $P(U \mid A) = 0.05$, $P(A) = 0.60$, $P(U \mid B) = 0.02$, $P(B) = 0.40$.

Want to find $P(A \mid U)$. By Bayes' Rule,

$$P(A \mid U) = \frac{P(U|A)P(A)}{P(U|A)P(A) + P(U|B)P(B)}.$$

$$P(A|U) = \frac{(0.05)(0.60)}{(0.05)(0.60) + (0.02)(0.40)} = 0.789.$$

Problem 2

A factory tests its machines for two common malfunctions. It finds that 15% of the machines have neither malfunction, 45% have Malfunction A, and 50% have Malfunction B (some have both malfunctions). If a machine is randomly chosen from the factory, find the probability distribution for Y, the number of malfunctions found in the machine.

SOLUTION

We are given $P(A) = 0.45$, $P(B) = 0.5$. Further, we are told that 15% of machines have neither. Note that $P(A \cup B) = 1 - P(neither) = 1 - 0.15 = 0.85$. An alternative way of finding this is to note that "neither" can be written as $A' \cap B'$, so $0.15 = P(A' \cap B') = P[(A \cup B)'] = 1 - P(A \cup B)$, which again implies $P(A \cup B) = 0.15$.

We can then use the addition rule to solve $P(A \cap B) = P(A) + P(B) - P(A \cup B) = 0.45 + 0.5 - 0.85 = 0.1$. Note that Y = 0 when neither malfunction occurs $(A' \cap B')$, Y = 2 when both malfunctions occur $(A \cap B)$, and $P(Y = 1) = 1 - P(Y = 0) - P(Y = 2)$. Therefore, the probability distribution is given below

Y	$P(Y = y)$
0	0.15
1	0.75
2	0.10