

Practice Exam 2

1. (18 points) The daily sales at a local bookstore that focuses on books in mathematics, statistics, and finance are normally distributed with mean \$1200 and standard deviation \$200. You may find the `pnorm()` and/or `qnorm()` output below to be useful in completing this problem.

- (a) Find the probability that the daily sales are between \$900 and \$1500.
- (b) Find the interval that contains the middle 40% of daily sales numbers.
- (c) Daily sales are independent from one day to the next. What is the probability that the daily sales are outside the interval (\$900,\$1500) at least twice in a five-day work week?

z	pnorm(z)
-2	0.023
-1.5	0.067
-1	0.159
-0.5	0.309
0.5	0.691
1	0.841
1.5	0.933
2	0.977

p	qnorm(p)
0.1	-1.282
0.2	-0.842
0.3	-0.524
0.4	-0.253
0.6	0.253
0.7	0.524
0.8	0.842
0.9	1.282

2. (18 points)

A factory produces light bulbs, each of which is defective with probability 0.12, independently of others.

- (a) What is the probability that at least 7 of the next 10 bulbs are **not defective**?
 - (b) Starting from now, what is the probability that the first defective bulb is the 8th one produced?
 - (c) What is the probability that the first defective bulb occurs at an **odd-numbered trial** (1st, 3rd, 5th, ...)?
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3. (12 points) The random variable X has probability density function

$$f(x) = \begin{cases} x^2, & -1 < x < 0, \\ cx, & 0 < x < 2, \\ 0, & \text{otherwise.} \end{cases}$$

- (a) Find the value of c that makes $f(x)$ a valid probability density function.
 - (b) Find the cumulative distribution function for X .
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4. (15 points) The side length S of a square is uniformly distributed on the interval $[2, 4]$.

- (a) Write down the probability density function for S .
 - (b) Find the expected value for the area $A = S^2$ of the square.
 - (c) Find the variance for the area $A = S^2$ of the square.
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5. (12 points) Let $Y \sim \text{Poisson}(\lambda)$

- (a) Use what you know about $E(Y)$ and $V(Y)$ to derive an expression for $E(Y^2)$
 - (b) Find $E(Y(Y - 1)(Y - 2))$
 - (c) Use your answers to parts a and b to find an expression for $E(Y^3)$. Your answer should be in terms of λ only.
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6. (10 points) The random variable X has moment generating function

$$m(t) = (0.5 + 0.5e^{2t})^2.$$

- (a) Find $E[X]$.
- (b) Find $V(X)$.

7. (15 points) The total medical claims each year for employees in a company is modeled with pdf $f(y) = 2(1 - y)$, $0 \leq y \leq 1$, where y is in \$100,000s.

- (a) Find the mean of the total claims
- (b) Find the 90th percentile of the distribution
- (c) If the company set aside \$75,000, what is the probability they cover all the claims for a year.

8. (4 points) A computer randomly generates a number Y that ranges from 100 to 150. Consider the following function:

$$h(y) = \begin{cases} 0, & y < 100 \\ 0.02y - 2, & 100 \leq y \leq 150 \\ 0, & y > 150 \end{cases}$$

The function $h(y)$ is _____.

- A) the cumulative distribution function (cdf) of Y
- B) the probability density function (pdf) of Y
- C) neither the cdf nor the pdf of Y

Justify your answer.