STAT 5700 – Practice Exam #1a

Instructions

You must **SHOW YOUR WORK** to receive full credit.

Problem 1 (20 points)

The random variable X has the following probability distribution:

x	2	4	10
$p(x)$	0.2	0.5	0.3

(a) Find the expected value of X.

(b) Find the variance and standard deviation of X.

(c) Find $E[X^3]$.

a) $E(x) = \sum x\, p(x) = 2(.2) + 4(.5) + 10(.3) = \boxed{5.4}$

b) $V(x) = E(x^2) - (E(x))^2$

$E(x^2) = \sum x^2 p(x) = 2^2(.2) + 4^2(.5) + 10^2(.3) = 38.8$

$V(x) = 38.8 - 5.4^2 = \boxed{9.64}$

$SD(x) = \sqrt{9.64} = \boxed{3.1}$

c) $E(x^3) = \sum x^3 p(x) = 2^3(.2) + 4^3(.5) + 10^3(.3) = \boxed{333.6}$

Problem 2 (24 points)

A factory produces light bulbs, each of which is defective with probability 0.12, independently of others.

(a) What is the probability that at least 7 of the next 10 bulbs are **not defective**?

(b) Starting from now, what is the probability that the first defective bulb is the 8th one produced?

(c) What is the probability that the first defective bulb occurs at an **odd-numbered trial** (1st, 3rd, 5th, ...)?

X = # of defective light bulbs $\quad X \sim \text{binom}(n, .12)$

a) at least 7 not defective $\Rightarrow$ at most 3 defective

$P(X \leq 3) = p(0) + p(1) + p(2) + p(3) = \boxed{0.976}$

$p(x) = \binom{10}{x}(.12)^x(.88)^{10-x}$

$p(0) = \binom{10}{0}.12^0(.88)^{10} = .2785$

$p(1) = \binom{10}{1}.12^1(.88)^9 = .3798$

$p(2) = \binom{10}{2}.12^2(.88)^7 = .2330$

$p(3) = \binom{10}{3}.12^3(.88)^6 = 0.0847$

b) $(.88)^7(.12) = 0.049$

c) Let Y = day the 1st defective arrives

y	$P(Y=y)$
1	$(.12)$
2	$(.88)(.12)$
3	$(.88)^2(.12)$
4	$(.88)^3(.12)$
5	$(.88)^4(.12)$
$\vdots$	

$p(y) = .12(.88)^{y-1}$

$P(1^{st}\text{ defective odd}) = P(Y = 2k+1), \quad k = 0, 1, \ldots$

$= \sum_{k=0}^{\infty}(.12)(.88)^{2k} = (.12)\underbrace{\sum_{k=0}^{\infty}(.88^2)^k}_{\text{geometric series}}$

$= (.12)\left[\frac{1}{1-.88^2}\right]$

$= \boxed{0.5319}$

Problem 3 (16 points)

A medical clinic refers patients to one of three doctors for checkups:

- Dr. A (50% of patients),
- Dr. B (30% of patients),
- Dr. C (20% of patients).

Let O = on time

$P(O|A) = 0.9$

$P(O|B) = 0.85$

$P(O|C) = 0.75$

The doctors complete the checkups on time with probabilities 0.9 (Dr. A), 0.85 (Dr. B), and 0.75 (Dr. C).

(a) What percentage of all checkups are completed on time?

(b) If a checkup was **not** completed on time, what is the probability that Dr. C was the doctor?

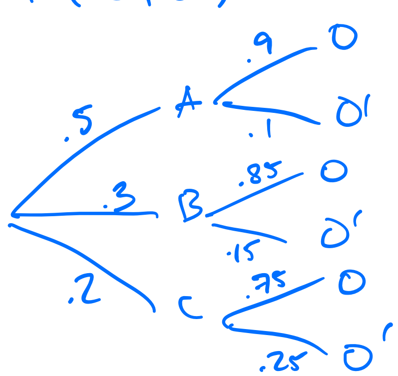
a) $P(O) = P(O \cap A) + P(O \cap B) + P(O \cap C)$

$= P(O|A)P(A) + P(O|B)P(B) + P(O|C)P(C)$

$= 0.9(.5) + 0.85(.3) + .75(.2)$

$= \boxed{0.855}$

b) $P(C|O') = \dfrac{P(C \cap O')}{P(O')} = \dfrac{(.2)(.25)}{1 - .855} = \boxed{0.345}$



Problem 4 (16 points)

A student organization has 20 members: 12 undergraduates and 8 graduate students. A committee of 4 members is chosen at random (without replacement).

(a) What is the probability that all 4 committee members are undergraduates?

(b) What is the probability that the committee has at least one undergraduate and at least one graduate student?

a) $$\frac{\binom{12}{4}\binom{8}{0}}{\binom{20}{4}} = \frac{\frac{12!}{8!\,4!}\;\frac{8!}{0!\,8!}}{\frac{20!}{16!\,4!}} = \frac{495}{4845} = \boxed{0.1022}$$

b) $$1 - [P(\text{all undergrad}) + P(\text{all grad})]$$

$$P(\text{all grad}) = \frac{\binom{12}{0}\binom{8}{4}}{\binom{20}{4}} = \frac{\frac{8!}{4!\,4!}}{4845} = \frac{70}{4845} = .0144$$

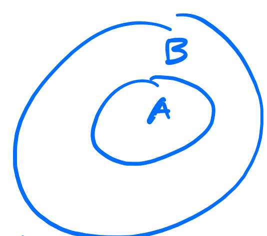
$$1 - .1022 - .0144 = \boxed{.8834}$$

Problem 5 (6 points)

Prove the following statement. If $A \subset B$, then $P(A|B) = \frac{P(A)}{P(B)}$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)}{P(B)} \checkmark$$

$A \cap B = A$ b/c $A \subset B$



Problem 6 (12 points)

Two fair coins are flipped. $S = \{HH, HT, TH, TT\}$

- Let A be the event that exactly one head is observed. $A = \{HT, TH\}$
- Let B be the event that the first coin shows heads. $B = \{HH, HT\}$

By checking an appropriate probability condition, determine whether A and B are independent.

check $P(A \cap B) \stackrel{?}{=} P(A)P(B)$

$A \cap B = \{HT\}$

$P(A \cap B) = \frac{1}{4}$

$P(A) = P(B) = \frac{2}{4}$

$P(A)P(B) = \frac{2}{4}\frac{2}{4} = \frac{4}{16} = \frac{1}{4} = P(A \cap B)$

A, B independent ✓

Multiple Choice (6 points)

Problem 7 (3 pts)

Suppose that a fair coin is flipped 10 times. Which is more likely – that the flips result in 5 heads and 5 tails, or that the flips result in 6 of one outcome and 4 of the other?

A. 5 of each
B. 6–4 split
C. These are equally likely
D. Not enough information to decide which of (A) or (B) is greater

Problem 8 (3 pts)

(S) and (T) are events with P(S) = 0.7 and P(T) = 0.6. Which of the following – A or B – is greater? Or are they equal? Or is there not enough information to decide?

A. 0.42
B. $P(S \cap T)$ C. 0.42 and $P(S \cap T)$ are exactly the same
D. There is not enough information to determine which of (A) or (B) is greater

need to know whether S, T are independent