HW 09 SOLUTIONS

Practice Problems

4.123a

$Y \sim \text{Beta}(\alpha = 4,\ \beta = 3)$, $k = \frac{\Gamma(4+3)}{\Gamma(4)\Gamma(3)} = 60$
95th percentile: $\phi_{.95} = 0.84684$

4.127

For $\alpha = \beta = 1,\ f(y) = \frac{\Gamma(2)}{\Gamma(1)\Gamma(1)} y^{1-1}(1-y)^{1-1} = 1 \Rightarrow Y \sim \text{Uniform}(0,1)$

4.165

The pdf of Y is in the form of a gamma density with $Y \sim \text{Gamma}(\alpha = 2,\ \beta = 0.5)$
(a) $c = \frac{1}{\Gamma(2)0.5^2} = 4$
(b) $E(Y) = \alpha\beta = 2(0.5) = 1$, $V(Y) = \alpha\beta^2 = 2(.5)^2 = 0.5$
(c) MGF: $M(t) = (1-\beta t)^{-\alpha} = (1-0.5t)^{-2},\ t < 2$

Submitted problems

4.126

(a) $\int_0^y (6t - 6t^2)dt = 3y^2 - 2y^3$, so $F(y) = \begin{cases} 0, & y < 0 \\ 3y^2 - 2y^3, & 0 \le y \le 1 \\ 1, & y > 1 \end{cases}$

(b) not shown

(c) $P(0.5 \le Y \le 0.8) = F(0.8) - F(0.5) = 1.92 - 1.092 - 0.75 + 0.25 = 0.396$

4.130

$\sigma^2 = E(Y^2) - \mu^2$,

$$E(Y^2) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \int_0^1 y^{\alpha+1}(1-y)^{\beta-1} dy = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \frac{\Gamma(\alpha+2)\Gamma(\beta)}{\Gamma(\alpha+2+\beta)} = \frac{(\alpha+1)\alpha}{(\alpha+\beta)(\alpha+\beta+1)}$$

$V(Y) = \frac{(\alpha+1)\alpha}{(\alpha+\beta)(\alpha+\beta+1)} - \left(\frac{\alpha}{\alpha+\beta}\right)^2 = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$

(b)

$$P(Y_2 \leq Y_1/2) = \int_0^1 \int_0^{y_1/2} 3y_1 \, dy_1 \, dy_2 = 0.5.$$