HW 08 SOLUTIONS

Practice Problems

4.59

(a) $z_0 = 0$

(b) $z_0 = 1.10$

(c) $z_0 = 1.645$

(d) $z_0 = 2.576$

4.73

$Y \sim N(950, 10^2)$

(a) $P(947 \leq Y \leq 958) = P(-0.3 \leq Z \leq 0.8) = 0.406$

(b) $P(Y \leq c) = 0.8531 \Rightarrow z_0 = 1.05$
$c = 950 + (1.05)(10) = 960.5$ mm

4.89

(a) Note that $\int_2^\infty \frac{1}{\beta} e^{-y/\beta} dy = e^{-2/\beta} = 0.0821 \Rightarrow \beta = 0.8$

(b) $P(Y \leq 1.7) = 1 - e^{-1.7/0.8} = 0.5075$

4.93

Let Y be the time between fatal airplane accidents. So $Y \sim \text{Exponential}(\beta = 44)$.
(a) $P(Y \leq 31) = \int_0^{31} \frac{1}{44} e^{-y/44} = 1 - e^{-31/44} = 0.5057$
(b) $V(Y) = \beta^2 = 44^2 = 1936$

4.109

$Y \sim \text{Gamma}(\alpha = 3,\ \beta = 2)$, $L = 30Y + 2Y^2$
$E(L) = 30E(Y) + 2E(Y^2) = 30(6) + 2(12 + 6^2) = 276$
$V(L) = E(L^2) - [E(L)]^2 = E(900Y^2 + 120Y^3 + 4Y^4) - 276^2$. $E(Y^3) = \int +0^{\infty} \frac{y^5}{16} e^{-y/2} = 480$ and $E(Y^4) = \int +0^{\infty} \frac{y^6}{16} e^{-y/2} = 5760$ Thus, $V(Y) = 900(48) + 120(480) + 4(5760) - 276^2 = 47664$

Submitted Problems

4.62

(a) $P(Z^2 < 1) = P(-1 < Z < 1) = 0.6826$

(b) $P(Z^2 < 3.84146) = P(-1.96 < Z < 1.96) = 0.95$

4.80

$A = L * W = |Y| * 3|Y| = 3Y^2$
$E(A) = 3E(Y^2) = 3(\sigma^2 + \mu^2)$

4.96abc

$Y \sim \text{Gamma}(\alpha = 4,\ \beta = 2)$ (a) $k = 1/(\Gamma(4)2^4) = 1/96$ (b) $Y \sim \chi^2$ with $\nu = 2\alpha = 8$ degrees of freedom (c) $E(Y) = \alpha\beta = 4(2) = 8$, $V(Y) = \alpha\beta^2 = 4(2^2) = 16$

4.104

$Y \sim \text{Exponential}(\beta = 100)$, $P(Y > 200) = e^{-2}$ Let the random variable X be the number of components that operate in the equipment for more than 200 hours. Then $X \sim \text{Binomial}(n = 3, p = e^{-2})$, $P(X \geq 2) = P(X = 2) + P(X = 3) = 3(e^{-2})^2(1 - e^{-2}) + (e^{-2})^3 = 0.05$

4.110

$Y \sim \text{Gamma}(\alpha = 3,\ \beta = 0.5)$, $E(Y) = 1.5$, $V(Y) = 0.75$