HW 04 SOLUTIONS

Practice Problems

3.15

a. $p(0) = P(Y = 0) = (.48)^3 = .1106$, $p(1) = P(Y = 1) = 3(.48)^2(.52) = .3594$, $p(2) = P(Y = 2) = 3(.48)(.52)^2 = .3894$, $p(3) = P(Y = 3) = (.52)^3 = .1406$.

b. The graph is omitted.

c. $P(Y = 1) = .3594$.

3.21

Note that $E(N) = E(8\pi R^2) = 8\pi E(R^2)$. So, $E(R^2) = 21^2(.05) + 22^2(.20) + \cdots + 26^2(.05) = 549.1$. Therefore $E(N) = 8\pi(549.1) = 13{,}800.388$.

3.31

a. The mean of W will be larger than the mean of Y if $\mu > 0$. If $\mu < 0$, the mean of W will be smaller than μ. If $\mu = 0$, the mean of W will equal μ.

b. $E(W) = E(2Y) = 2E(Y) = 2\mu$.

c. The variance of W will be larger than σ^2, since the spread of values of W has increased.

d. $V(X) = E[(X - E(X))^2] = E[(2Y - 2\mu)^2] = 4E[(Y - \mu)^2] = 4\sigma^2$.

3.37

a. Not a binomial random variable.

b. Not a binomial random variable.

c. Binomial with $n = 100$, $p =$ proportion of high school students who scored above 1026.

d. Not a binomial random variable (not discrete).

e. Not binomial, since the sample was not selected among all female HS grads.

3.39

Let $Y =$ number of components failing in less than 1000 hours. Then $Y \sim Binomial(n = 4, p = .2)$.
a. $P(Y = 2) = \binom{4}{2}(.2)^2(.8)^2 = 0.1536$.
b. The system operates if 0, 1, or 2 components fail. $P(\text{system operates}) = .4096 + .4096 + .1536 = .9728$.

3.51

a. $P(\text{at least one 6 in four rolls}) = 1 - P(\text{no 6's in four rolls}) = 1 - (5/6)^4 = 0.51775$.

b. In a single toss of two dice, $P(\text{double 6}) = 1/36$. Then $P(\text{at least one double 6 in 24 rolls}) = 1 - P(\text{no double 6's in 24 rolls}) = 1 - (35/36)^{24} = 0.4914$.

3.55

We use the identity

$$\begin{aligned}
E[Y(Y-1)(Y-2)] &= \sum_{y=0}^{n} \frac{y(y-1)(y-2)n!}{y!(n-y)!} p^y (1-p)^{n-y} \\
&= \sum_{y=3}^{n} \frac{n(n-1)(n-2)(n-3)!}{(y-3)!(n-3-(y-3))!} p^y (1-p)^{n-y} \\
&= n(n-1)(n-2)p^3 \sum_{z=0}^{n-3} \binom{n-3}{z} p^z (1-p)^{n-3-z} \\
&= n(n-1)(n-2)p^3
\end{aligned}$$

Equating this to $E(Y^3) - 3E(Y^2) + 2E(Y)$, it is found that $E(Y^3) = 3n(n-1)p^2 - n(n-1)(n-2)p^3 + np$.

3.59

If Y = number of defective motors, then $Y \sim Binomial(n = 10, p = .08)$. Then $E(Y) = .8$. The seller's expected net gain is $1000–200E(Y) = 840$.

Submitted Problems

3.12

$E(Y) = 1(.4) + 2(.3) + 3(.2) + 4(.1) = 2.0$

$E(1/Y) = 1(.4) + \frac{1}{2}(.3) + \frac{1}{3}(.2) + \frac{1}{4}(.1) = 0.6417$

$E(Y^2 - 1) = E(Y^2) - 1 = [1^2(.4) + 2^2(.3) + 3^2(.2) + 4^2(.1)] - 1 = 5 - 1 = 4$

$V(Y) = E(Y^2) - [E(Y)]^2 = 5 - 2^2 = 1$

3.20

With probability 0.3 the volume is $8 \cdot 10 \cdot 30 = 2400$.
With probability 0.7 the volume is $8 \cdot 10 \cdot 40 = 3200$.
Mean $= 0.3(2400) + 0.7(3200) = 2960$

3.22

$p(y) = P(Y = y) = 1/6$ for $y = 1, 2, \ldots, 6$.
So $E(Y) = 3.5$ and $V(Y) = 2.9167$.

3.24

Distribution of Y = number of bottles with serious flaws:

y	0	1	2
$p(y)$	0.81	0.18	0.01

Thus $E(Y) = 0(.81) + 1(.18) + 2(.01) = 0.20$ and
$V(Y) = 0^2(.81) + 1^2(.18) + 2^2(.01) - (0.20)^2 = 0.18$

3.30

a. Mean of X is larger than mean of Y.

b. $E(X) = E(Y + 1) = E(Y) + 1 = \mu + 1$.

c. Variances of X and Y are the same.

d. $V(X) = E[(X - E(X))^2] = E[(Y + 1 - \mu - 1)^2] = E[(Y - \mu)^2] = \sigma^2$

3.44

Let Y = number of successful operations, $Y \sim \text{Binomial}(n = 5, p)$

- With $p = 0.8$: $P(Y = 5) = 0.8^5 = 0.328$
- With $p = 0.6$: $P(Y = 4) = 5(0.6^4)(0.4) = 0.259$
- With $p = 0.3$: $P(Y < 2) = P(Y = 1) + P(Y = 0) = 0.528$

3.58

If $Y \sim \text{Binomial}(n = 4, p = 0.1)$:

$E(Y) = 0.4$, $V(Y) = 0.36$, so $E(Y^2) = 0.36 + (0.4)^2 = 0.52$

Then $E(C) = 3(0.52) + (0.36) + 2 = 3.96$