HW 03 SOLUTIONS

Practice Problems

2.125

Define the events: D: has the disease, H: test indicates the disease. Thus, $P(H|D) = 0.9$, and $P(H'|D') = 0.9$, $P(D) = 0.01$, $P(D') = 0.99$. Then by Bayes' rule, $P(D|H) = \frac{P(H|D)P(D)}{P(H|D)P(D)+P(H|D')P(D')} = \frac{0.9(0.01)}{0.9(0.01)+0.01(0.99)} = \frac{1}{12}$.

2.133

Define the events: G: student guesses, C: student is correct. Then $P(G'|C) = \frac{P(C|G')P(G')}{P(C|G')P(G')+P(C|G)P(G)} = 0.9412$.

2.137

Let $A = \{\text{both balls are white}\}$ and for $i = 1, 2, \ldots, 5$, let $A_i =$ "both balls selected from bowl i are white." Then $\bigcup A_i = A$. Also let $B_i =$ "bowl i is selected." Then $P(B_i) = 0.2$ for all i.
a. $P(A) = \sum_{i=1}^{5} P(A|B_i)P(B_i) = \frac{1}{5}[0 + \frac{2}{5}\frac{1}{4} + \frac{3}{5}\frac{2}{4} + \frac{4}{5}\frac{3}{4} + 1] = 2/5$
b. By Bayes' rule, $P(B_3|A) = (3/50)/(2/50) = 3/20$.

3.1

$P(Y = 0) = P(\text{no impurities}) = 0.2$, $P(Y = 1) = P(\text{exactly one impurity}) = 0.7$, $P(Y = 2) = 0.1$.

3.3

$p(2) = P(DD) = 1/6$, $p(3) = P(DGD) + P(GDD) = 2(2/4)(2/3)(1/2) = 2/6$, $p(4) = P(GGDD) + P(DGGD) + P(GDGD) = 3(2/4)(1/3)(1) = 1/2$.

3.5

There are $3! = 6$ possible ways to assign the words to the pictures. Of these, one is a perfect match, three have one match, and two have zero matches. Thus, $p(0) = 2/6$, $p(1) = 3/6$, $p(3) = 1/6$.

Submitted Problems

2.124

Define the events: D: democrat, R: republican, F: favors issue. Then $P(D|F) = \frac{P(F|D)P(D)}{P(F|D)P(D)+P(F|R)P(R)} = \frac{(0.7)(0.6)}{(0.7)(0.6)+(0.3)(0.4)} = 7/9$.

2.130

Define the events: C: contract lung cancer, S: worked in a shipyard. Given $P(S|C) = 0.22$, $P(S|C') = 0.14$, and $P(C) = 0.0004$. By Bayes' rule, $P(C|S) = \frac{P(S|C)P(C)}{P(S|C)P(C)+P(S|C')P(C')} = \frac{(0.22)(0.0004)}{(0.22)(0.0004)+(0.14)(0.9996)} \approx 0.0006$.

2.132

For $i = 1, 2, 3$, let F_i = plane is found in region i, N_i = not found in region i, R_i = plane is in region i. Then $P(F_i|R_i) = 1 - \alpha_i$ and $P(R_i) = 1/3$.
a. $P(R_1|N_1) = \frac{P(N_1|R_1)P(R_1)}{P(N_1|R_1)P(R_1)+P(N_1|R_2)P(R_2)+P(N_1|R_3)P(R_3)} = \frac{\alpha_1(1/3)}{\alpha_1(1/3)+(1/3)+(1/3)} = \frac{\alpha_1}{\alpha_1+2}$.
b. $P(R_2|N_1) = \frac{1/3}{\alpha_1/3+1/3+1/3} = \frac{1}{\alpha_1+2}$.
c. $P(R_3|N_1) = \frac{1}{\alpha_1+2}$.

2.143

Since $P(B) = P(B \cap A) + P(B \cap A')$, dividing each part by $P(B)$ gives $1 = \frac{P(B \cap A)}{P(B)} + \frac{P(B \cap A')}{P(B)} = P(A|B) + P(A'|B)$

2.156

a.
   i. $1 - 5686/97900 = 0.942$.
   ii. $(97900 - 43354)/97900 = 0.557$.
   iii. $10560/14113 = 0.748$.
   iv. $(646 + 375 + 568)/11533 = 0.138$.

b. If the US population in 2002 was known, this could be used to divide into the total number of deaths in 2002 to give a probability.

2.172

Only $P(A|B) + P(A'|B) = 1$ is true for any events A and B.

3.2

We know $P(HH) = P(TT) = P(HT) = P(TH) = 0.25$. So $P(Y = -1) = 0.5$, $P(Y = 1) = 0.25$, $P(Y = 2) = 0.25$.

3.6

There are 10 sample points, all equally likely: $(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 4), (2, 5), (3, 4), (3, 5), (4, 5)$.
a. $p(2) = 0.1$, $p(3) = 0.2$, $p(4) = 0.3$, $p(5) = 0.4$.
b. $p(3) = 0.1$, $p(4) = 0.1$, $p(5) = 0.2$, $p(6) = 0.2$, $p(7) = 0.2$, $p(8) = 0.1$, $p(9) = 0.1$.

3.10

Let R = rental on a given day, N = no rental. Thus the sequence of interest is $RR, RNR, RNNR, RNNNR, \ldots$. Consider the position immediately following the first R: it is filled by an R with prob 0.2, N with prob 0.8. Thus, $P(Y = 0) = 0.2$, $P(Y = 1) = 0.8(0.2) = 0.16$, $P(Y = 2) = 0.128$, etc. In general, $P(Y = y) = 0.2(0.8)^y$, $y = 0, 1, 2, \ldots$