HW 01 SOLUTIONS

Practice Problems

2.1

$A = \{FF\},\ B = \{MM\},\ C = \{MF, FM, MM\}.\ A \cap B = \emptyset,\ B \cap C = \{MM\},\ C \setminus B = \{MF, FM\},\ A \cup B = \{FF, MM\},\ A \cup C = S,\ B \cup C = C.$

2.9

$S = \{A^+, B^+, AB^+, O^+, A^-, B^-, AB^-, O^-\}$

2.15

a. Since the events are mutually exclusive,
   $P(S) = P(E_1) + \cdots + P(E_4) = 1.$
   So, $P(E_2) = 1 - 0.01 - 0.09 - 0.81 = 0.09.$

b. $P(\text{at least one hit}) = P(E_1) + P(E_2) + P(E_3) = 0.19.$

2.17

Let B = bushing defect, SH = shaft defect.

a. $P(B) = 0.06 + 0.02 = 0.08$

b. $P(B \text{ or } SH) = 0.06 + 0.08 + 0.02 = 0.16$

c. $P(\text{exactly one defect}) = 0.06 + 0.08 = 0.14$

d. $P(\text{neither defect}) = 1 - P(B \text{ or } SH) = 1 - 0.16 = 0.84$

2.19

a. $(V_1, V_1), (V_1, V_2), (V_1, V_3), (V_2, V_1), (V_2, V_2), (V_2, V_3), (V_3, V_1), (V_3, V_2), (V_3, V_3)$

b. If equally likely, all have probability 1/9.

c. $A = \{\text{same vendor gets both}\} = \{(V_1, V_1), (V_2, V_2), (V_3, V_3)\}$
$B = \{\text{at least one } V_2\} = \{(V_1, V_2), (V_2, V_1), (V_2, V_2), (V_2, V_3), (V_3, V_2)\}$ So, $P(A) = 1/3$, $P(B) = 5/9$,
$P(A \cup B) = 7/9$, $P(A \cap B) = 1/9$.

2.25

Unless exactly 1/2 of all cars in the lot are Volkswagens, the claim is not true.

2.31

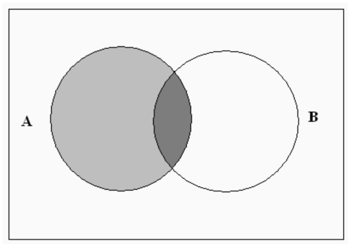
a. There are four "good" systems and two "defective" systems.
If two out of the six systems are chosen randomly, there are 15 possible unique pairs. Denoting the systems as $g_1, g_2, g_3, g_4, d_1, d_2$, the sample space is $S = \{g_1g_2, g_1g_3, g_1g_4, g_1d_1, g_1d_2, g_2g_3, g_2g_4, g_2d_1, g_2d_2, g_3g_4, g_3d_1, g_3d_2, g_4g_1, g_4d_1, d_1d_2\}$.
Thus: $P(\text{at least one defective}) = 9/15$, $P(\text{both defective}) = P(d_1d_2) = 1/15$.

b. If four are defective: $P(\text{at least one defective}) = 14/15$, $P(\text{both defective}) = 6/15$.

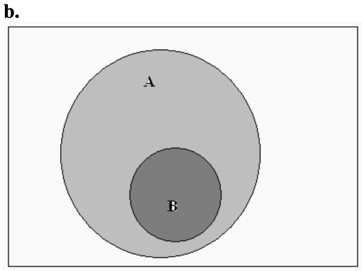
Submission Problems

2.4

2.4 a.



b.



2.8

a. $36 + 6 = 42$

b. 33

c. 18

2.14

a. $P(\text{needs glasses}) = 0.44 + 0.14 = 0.48$
b. $P(\text{needs glasses but doesn't use them}) = 0.14$
c. $P(\text{uses glasses}) = 0.44 + 0.02 = 0.46$

2.26

a. Let N_1, N_2 denote the empty cans and W_1, W_2 denote the cans filled with water.
Thus, $S = \{N_1N_2, N_1W_2, N_2W_2, N_1W_1, N_2W_1, W_1W_2\}$.

b. If this is merely a guess, the events are equally likely.
So, $P(W_1W_2) = 1/6$.

2.28

a. Denote the four candidates as A_1, A_2, A_3, and M.
Since order is not important, the outcomes are
$\{A_1A_2, A_1A_3, A_1M, A_2A_3, A_2M, A_3M\}$.

b. Assuming equally likely outcomes, all have probability 1/6.

c. $P(\text{minority hired}) = P(A_1M) + P(A_2M) + P(A_3M) = 0.5$

2.30

a. Let w_1 denote the first wine, w_2 the second, and w_3 the third.
Each sample point is an ordered triple indicating the ranking.

b. Triples:
$(w_1, w_2, w_3), (w_1, w_3, w_2), (w_2, w_1, w_3), (w_2, w_3, w_1), (w_3, w_1, w_2), (w_3, w_2, w_1)$

c. For each wine, there are 4 ordered triples where it is not last.
So, the probability is 2/3.

Additional Problem

Show that

a) $P(A_1 \cup A_2) \leq P(A_1) + P(A_2)$

b) $P(A_1 \cap A_2) \geq P(A_1) + P(A_2) - 1$

Solution

Both parts follow from the fact that $P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2)$.

Part a

$P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2) \leq P(A_1) + P(A_2)$ because $P(A_1 \cap A_2) \geq 0$.

Part b

$P(A_1 \cup A_2) \leq 1 \implies P(A_1) + P(A_2) - P(A_1 \cap A_2) \leq 1 \implies P(A_1) + P(A_2) - 1 \leq P(A_1 \cap A_2)$